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PAPER

Feedback-controlled and digitally processed coherent population trapping resonance conversion in ⁸⁷Rb vapour to high-contrast resonant peak

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Abstract

This work reports on the study of a new approach to achievement of high-contrast resonant signals from coherent population trapping (CPT) resonances in 87 Rb vapour based on feedback control and real-time digital processing of several measured parameters. This method consists in stabilisation of the value of a function depending on several system parameters measured as the frequency difference of the bichromatic pump radiation is scanned through adjustment of the pumping radiation power with a feedback loop. The present work made use of two such parameters: the pumping radiation power incident on and exiting from the optical cell. Exploration of the proposed method has shown that stabilisation of a linear combination of these two parameters results in a resonant peak whose contrast exceeds that of regular CPT resonance by more than two orders of magnitude at relatively slow CPT resonance scan rates, (scanning frequency of the frequency difference of the bichromatic field ~ 1 Hz). When dynamically exciting the CPT resonance (the scan frequency of the frequency difference of the bichromatic field equal to 2 kHz), the resonant peak contrast was enhanced by over an order of magnitude.

1. Introduction

Coherent population trapping (CPT) resonance in alkali metal vapours is often used in small-scale frequency standards [1, 2], highly sensitive magnetometers [3], and in other high-precision devices [4]. The resonance contrast, or the ratio of its amplitude to the baseline transmission (this definition of contrast allows its value to exceed 100%), is one of the key parameters of CPT resonance affecting stability and/or accuracy of devices based on it. In conventional configurations for excitation of a CPT resonance [5], the resonance contrast defined as the ratio of CPT resonance amplitude to the background is low and ranges from a fraction of a per cent [6] to several per cent [7–9]. Various methods were proposed that allow CPT resonance contrast to reach tens [10–15] and hundreds per cent [16–18]. However, all these methods are either only applicable in special cases or relatively complicated in technical implementation. For example, a polarisation-selective detection method [10] is used that requires analysis of the polarisation state of radiation passed through the optical cell. A different technique [11, 12] is only applicable in a special case when the levels of the Λ -system have definite values of the angular momenta. A push-pull method [8, 13] relies on orthogonally polarised radiation components with a temporal delay between them. Another approach [19] is based on detection of forward-scattered radiation rather than the radiation directly passed through the optical cell [20], therefore demanding registration of comparatively weak scattered signal (in most of the mentioned methods, the input power of pump radiation does not exceed 1 mW). Yudin et al [16] have reported 260% contrast in feedback-spectroscopy method where rubidium atomic fluorescence was measured when scanning the frequency difference of a bichromatic pumping field around the ground-state hyperfine-splitting frequency of ⁸⁷Rb, while a feedback loop was used to adjust the pumping radiation intensity in order to maintain a constant level of atomic vapour fluorescence. Contrast exceeding 760% was achieved through the crossed polarisers method [17]. Unfortunately, though, this method relies on

resonant Faraday rotation at CPT resonance, and significant contrast enhancement is only possible if the optical cell is placed within a relatively strong and highly stable stationary magnetic field in order to ensure a stable CPT resonance amplitude. Quite obviously, CPT resonances obtained with this technique are incompatible with high-sensitivity magnetometry. As far as we know, the method proposed by Shah *et al* [18] resulted in the record-highest CPT resonance contrast value of 900%. But at the same time, it is based on one of the most technically challenging approaches relying on four-wave mixing in rubidium atoms of radiation from two pump lasers.

It is pertinent to note that the feedback-based method [16] has until this moment remained the only one using feedback to enhance the contrast of a CPT resonance. One of its benefits comes from the possibility of relatively high CPT resonance contrast at better signal-to-noise ratio (SNR) and narrower resonance width because of nonlinear properties of interaction between the bichromatic pump radiation and the atomic three-level system. The principal drawback to the method is that relatively weak fluorescence radiation has to be measured, thus leading to relatively low SNR, especially manifest at relatively low pump radiation power (<1 mW) and small cell sizes. It is possible to improve the SNR of this fluorescence measurement by using photodetectors with large active area, higher pump power, bigger optical cells, etc. These arrangements, however, substantially reduce the range of applications compatible with the method, which, in effect, is a special case of the general method proposed and studied in the present work.

The general method of feedback-controlled and digitally processed CPT resonance conversion to a high-contract resonant peak, for the first time proposed and explored in the present work, consists in stabilisation of the value of a function depending on system parameters measured as the frequency difference of the bichromatic pump radiation is scanned through automatic adjustment of the pumping radiation power. In this work, we tested one of the most technically straightforward implementations of the proposed method where two system parameters—pump radiation power entering and exiting the optical cell—are measured and a linear combination thereof is stabilised. A resonant peak is then formed in the dependence of the pump radiation intensity upon the frequency difference of a bichromatic pumping field. The following discussion details the results obtained with this method both at low CPT resonance scanning frequencies (≤ 1 Hz) and under dynamic CPT resonance excitation in the scanning frequency range around 2 kHz.

2. Method

The proposed method is founded on the idea of conversion of the initial low-contrast CPT resonance into a sharp high-contract resonant peak through application of digital processing techniques within the feedback loop controlling the pumping radiation power. The pump radiation power is adjusted automatically so as to keep constant the value of a certain pre-set function of several system parameters measured during scanning of the frequency difference of the bichromatic pump radiation. In the process, the original CPT resonance is transformed into a resonant dependence of the pumping radiation power upon the frequency detuning or, as it is referred to in this work, a resonant peak.

Inclusion of the pump radiation power both entering and exiting the optical cell into the feedback parameters yields information about radiation absorption and transmission in the cell. Direct radiation measurement (as opposed to fluorescence or scattered radiation) generates signals with comparatively high SNR even from a low-power vertical-cavity surface-emitting laser. Nearly instantaneous calculation of the chosen function within the feedback loop from several measured signals is possible with fast analogue-to-digital (AD) and digital-to-analogue (DA) converters and rapid digital signal processing. AD/DA converters with sampling rates of 50 kS s⁻¹ and 24 bit precision are no longer exotic components and simultaneously provide a relatively broad feedback loop bandwidth (up to 50 kHz) and high SNR (over 70 dB). Application of real-time digital signal processing to the feedback loop leading to improved contrast and SNR of the resonant signal opens up fundamentally new possibilities of resonant peak formation on the basis of the CPT resonance profile.

A serious amount of research is necessary to reveal the entire broad scope and significance of the potential held by the proposed general method for generation of resonant peaks from CPT resonance profile. Therefore, in this first treatment of the subject, we will limit ourselves to a demonstration of significant improvement in the resonant peak contrast compared to the initial CPT resonance by way of stabilising the value of a linear combination of the two measured parameters—power of the radiation absorbed in the cell $P_{\rm abs}$ and pump power $P_{\rm pump}$:

$$F(P_{\text{abs}}, P_{\text{pump}}) = P_{\text{abs}} + S \times P_{\text{pump}} = \text{Const},$$
 (1)

where S is a constant experimentally chosen so as to result in the best resonant peak contrast. The parameter Const equals the value of function $F(P_{abs}, P_{pump})$ at the highest pump power $P_{pump} = P_{max}$ and zero frequency detuning. Figure 1 explains the principle of the studied method. Stabilisation of function (1), as the frequency is scanned, leads to reduction in the power of radiation transmitted through the optical cell down to value P_{min}^* ,

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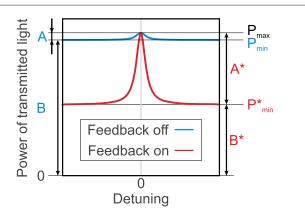


Figure 1. Dependence of the pump radiation power transmitted through the optical cell upon the frequency detuning without feedback and at constant pump power (blue curve) and with feedback loop active when the pump power is reduced as the frequency is detuned away from the resonance (red curve). A—CPT resonance amplitude and B—background without feedback; A^* and B^* —the corresponding amplitude and background with feed-back. P_{\max} —highest power of the radiation transmitted through the cell, P_{\min} , P_{\min}^* —minimal level of the radiation transmitted through the cell.

whereas the CPT resonance contrast is enhanced from the original value of C = A/B to $C^* = A^*/B^*$. It can be seen that the contrast is enhanced due to reduction of the pump radiation power and that the contrast is higher for lower P_{\min}^* . In other words, the larger is the excursion of the pump radiation power during scanning, the higher is the resonance contrast.

Let us estimate the value of S, using equation (1) at the maximum of the CPT resonance and in its vicinity. The power of radiation absorbed within the cell P_{abs} depends on P_{pump} as $P_{abs} = K \times P_{pump}$, where K is the coefficient of radiation absorption in the cell. In the vicinity of the CPT resonance, absorption coefficient K equals baseline absorption coefficient (K_{BL}), while at maximum of the CPT resonance, $K = K_{BL} - K_{CPT}$, where K_{CPT} is a coefficient taking into account reduced radiation absorption at maximum of the CPT resonance due to CPT. Respectively, at maximum of the CPT resonance and its vicinity, equation (1) takes the form:

$$(K_{\rm BL} - K_{\rm CPT}) \times P_{\rm pump} + S \times P_{\rm pump} = {\rm Const},$$
 (2)

where $P_{\text{pump}} = P_{\text{max}}/(1 - K) = P_{\text{max}}/(1 - K_{\text{BL}} + K_{\text{CPT}})$, and

$$K_{\rm BL} \times P_{\rm pump} + S \times P_{\rm pump} = {\rm Const},$$
 (3)

where $P_{\text{pump}} = P_{\text{min}}^*/(1 - K) = P_{\text{min}}^*/(1 - K_{\text{BL}})$. In approximation of $K_{\text{BL}} \gg K_{\text{CPT}}$, it follows from equations (2) and (3) that $S \approx -K_{\text{BL}}$.

3. Experiment

The experimental installation used in studies of the proposed method is schematically shown in figure 2. An optical cell with approximate volume of 1 cm³ and an anti-relaxation coating applied to the walls was filled with ⁸⁷Rb vapour and placed inside a three-layer Mu-metal shield, the cell temperature being stabilised around 60 °C to the precision of 1 mK. Bichromatic pump radiation with a specific frequency difference (6.835 GHz) of two near-resonant optical fields was provided by a laser diode (vertical-cavity surface-emitting laser) whose injection current was modulated at the frequency equal to half the desired frequency difference (3.417 GHz). This led to formation of first-order side peaks around the laser carrier frequency in the output radiation spectrum. The input power of the RF signal (3.417 GHz) was chosen from the condition of creating such side-band amplitude, at which the CPT resonance light shift was minimised [21–23]. The pumping radiation was guided into the cell through an amplitude electro-optical modulator (EOM), which could be used to adjust the input pumping radiation power (0.5 mW at 795 nm within a line width of \sim 100 MHz, with beam diameter of 2 mm). All the electronic components of the experimental installation were controlled through National Instruments PXI platform [24]. For measurement of pump power before and after the cell, two photodetectors were used (PD_1 and PD_2 in figure 2). The chosen placement of photo-detectors allows independent measurement of $P_{abs} = PD_1 - PD_2$ and $P_{pump} = PD_2$, where PD_1 and PD_2 are signals from the respective photo-detectors. Photodetector PD₂ received 50% of the pump radiation. The signals of photodetectors PD₁ and PD₂ were digitised by two 24 bit ADC's at sampling rate of 50 kS s⁻¹. Further signal processing was carried out in a field-programmable gate array, which generated an error signal at the output.

This signal was converted to the analogue form with a 16 bit DAC at the same sampling rate of $50 \, \text{kS s}^{-1}$,

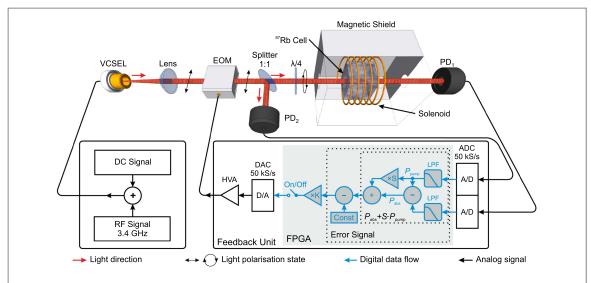
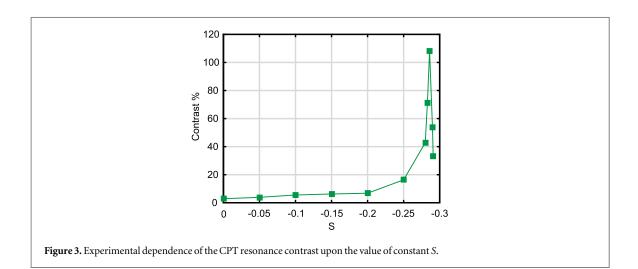


Figure 2. Schematic diagram of the experimental set-up for testing and study of the proposed method for achievement of high-contrast resonant signals from CPT resonances: EOM—amplitude electro-optical modulator, PD₁, PD₂—photodetectors, HVA—high-voltage amplifier, LPF—digital low-pass filter with cut-off frequency 10 kHz, K—feedback loop gain, FPGA—field-programmable gate array, A/D—analogue-to-digital converter, D/A—digital-to-analogue converter.



amplified, and then used to control the EOM. Gain *K* of the feedback loop was slightly below the value at which negative feedback became positive.

Scanning the frequency difference of the bichromatic pumping field around the resonant value modifies $P_{\rm abs}$ and, respectively, the difference between signals from the photodetectors. The value of $P_{\rm abs}$ or that of a certain function related to $P_{\rm abs}$ can be then stabilised by corresponding adjustment of the input pumping power with the EOM. A resonant peak in this case will be seen in the dependence of the pump radiation intensity at the exit from the EOM upon the frequency difference of the bichromatic pump field. For automatic stabilisation, we used a feedback system controlling the transmission of the EOM with an error signal obtained by digital processing of photo-detector signals.

It should be noted that in the proposed method, a resonant peak is formed in the dependence of the pump field intensity upon the frequency difference of the bichromatic pump field and may be registered by any of the system photodetectors. We used PD_1 because the resonance signal from this photodetector had more contrast.

When $P_{\rm abs}$ value alone was stabilised (which corresponds to the method described earlier [16]), the resonant peak contrast improved to 3% compared to 1% for the initial CPT resonance. Stabilisation of the combination of parameters $P_{\rm abs}$ and $P_{\rm pump}$ given in equation (1) resulted in a dramatically higher resonant peak contrast enhancement from 1% to 108% at the value of coefficient S = -0.28. Figure 3 contains an experimental dependence of the resonant peak contrast upon coefficient S. There is a very pronounced maximum at negative S values around S = -0.28. The experimentally found optimal value of constant S approximately matches the fraction of the pump radiation absorbed in the optical cell off the CPT resonance (0.3). In other words, it is close to the baseline absorption coefficient, thus agreeing with our estimation of the optimal value of S presented

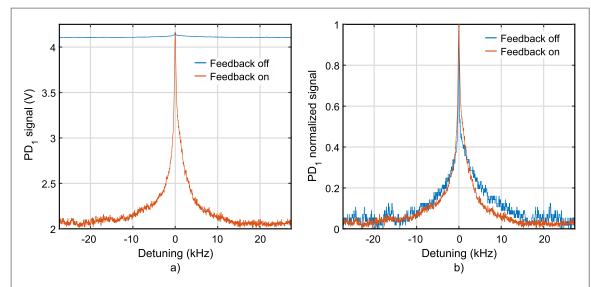


Figure 4. Signals from photodetector PD_1 without (blue curve) and with (red curve) the feedback control and digital processing at low resonance scanning frequency ($\leqslant 1$ Hz): (a)—unprocessed signals, (b)—the CPT resonance on the blue curve normalised to the amplitude of the 'red' resonant peak. Detuning is the detuning of the frequency difference between the components of the bichromatic pump radiation from the value of 6.835 GHz corresponding to the transition frequency between the sub-levels of hyperfine ⁸⁷Rb ground level splitting.

earlier. This means that the highest resonant peak contrast is achieved when only CPT resonance absorption is stabilised (and baseline absorption is eliminated from the loop).

Figure 4 demonstrates resonances registered with photodetector PD₁ without (upper blue curve—raw CPT resonance) and with (red curve—generated resonant peak) feedback at low resonance scanning frequency (\leq 1 Hz), while stabilising the value of function (1).

It is evident from figure 4 that a barely discernible raw CPT resonance on the blue curve (no back-coupling) turns into a greatly more pronounced resonant peak on the red curve when the feedback loop is closed. Thus, the resonant peak contrast became more than 100 times better than that of the initial CPT resonance (from 1% to 108%). In order to find out whether or not the generated resonant peak is broadened in comparison to the raw CPT resonance, we normalised the amplitude of the initial CPT resonance to that of the generated resonant peak, see figure 4(b). It can be seen that the width of these resonances (~500 Hz) is virtually identical, which means that considerable enhancement of the resonant peak contrast in the proposed method does not lead to peak broadening. It can be also seen from this figure that the SNR of the high-contract resonant peak generated via the proposed feed-back method with digital processing of the error signal is approximately twice as high as the SNR or the initial CPT resonance. It must be pointed out that the question of the exact mechanism that leads to SNR improvement and the potential of this improvement requires additional research. It is possible that the observed improvement in SNR is related to stabilisation of the pump radiation power by the feedback system. Additionally, SNR may also be improved during the nonlinear process of the proposed contrast enhancement method.

At high CPT resonance scanning frequency, the resonant peak contrast obtained with the back-coupling active was similarly improved, but the enhancement factor in this case was considerably lower, not exceeding 25. This is probably due to a considerable deformation of dynamically excited CPT resonant line profile with simultaneous degradation of the SNR [25]. The corresponding resonances without (blue curve) and with (red curve) feedback at scanning frequency of 2 kHz are given in figure 5.

The method proposed in the present work is considerably more universal and technically simple to implement than proposed earlier [16] because it does not need:

- a photodetector registering the intensity of Rb vapour fluorescence in the immediate vicinity of the optical cell, which may affect the distribution of the magnetic field within the cell;
- suppression of stray radiation (pump radiation reflected and scattered from the cell walls) leaking onto the photodetector;
- photodetectors with large active area, which are relatively slower and thus limit the frequency bandwidth of the feedback loop.

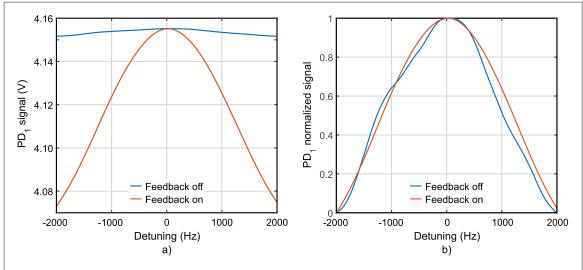


Figure 5. Signals from photodetector PD_1 with the feedback loop open (blue curve) and closed (red curve) at dynamic CPT resonance excitation (scanning frequency \sim 2 kHz): (a)—unprocessed signals, (b)—the CPT resonance on the blue curve normalised to the amplitude of the 'red' resonant peak.

It is further worth noting that the proposed method is applicable not only to contrast enhancement of CPT resonances in cells with anti-relaxation coating, but also in many other cases. For instance, it could be CPT resonances in cells with buffer gas, excitation of double radio-optical resonances [26], and even pulsed resonance excitation [27, 28] when using a sufficiently fast feed-back system. It should be also pointed out that the proposed technique of contrast enhancement of a resonant spectroscopic signal may be suitable not only for metrological and sensing applications, but also for fundamental and other research.

Additionally, when applying the proposed contrast-enhanced CPT resonance technique, for example in frequency standards, the feedback loop controlling the pump radiation power may operate in parallel with a second loop stabilising the frequency of the quartz oscillator. Successful co-existence of several feedback loops combined with the one controlling the pump radiation power requires, first of all, that the power of radiation exiting the cell not fall below the level measurable by photodetector PD_1 . This situation corresponds to the EOM completely cutting off the pump radiation and making the feedback loop depending on the pump radiation power disabled. Correspondingly, the value of P_{\min}^* may not fall below a certain threshold, thus limiting the possible improvement of the CPT resonance contrast in the proposed method at fixed pump radiation power. Further contrast enhancement may be possible at higher available pump radiation power, as well as with more sensitive photodetector PD_1 .

4. Conclusion

In the present work, we proposed and studied a new method of resonant peak generation from the CPT resonance profile based on stabilisation of the value of a function of several system parameters measured as the frequency difference of the bichromatic pump radiation is scanned through automatic adjustment of the pumping radiation power with a feedback loop. This method provides efficient transformation of the original CPT resonance signal into a high-contrast resonant peak under both quasi-stationary and dynamic CPT resonance excitation at relatively low pump radiation power (0.5 mW) and compact optical cell dimensions $(1 \text{ cm}^3 \text{ volume})$. Stabilisation of the value of a linear combination of the measured power of the radiation absorbed in the cell and the pump power results in a 25-fold contrast enhancement (from 0.07% to 1.8%) for dynamically excited CPT resonances and more than two orders of magnitude better contrast (from 1% to 108%) of quasi-stationarily excited resonances while not introducing any additional broadening to the resonant signals. Moreover, the proposed method has a significant potential for modification of the resonant peak shape because of the freedom to define the function $F(P_{abs}, P_{pump})$ with mathematical methods.

It should be pointed out that although digital processing of the raw CPT resonance signal in the absence of feedback may also improve CPT resonance contrast, it does not affect the SNR of the resonant signal. Conversely, a resonant peak generated with the aid of nonlinear feedback control features a better quality factor of the CPT resonance (ratio of the resonance contrast to the resonance width) [29] and a two-fold SNR improvement compared to the original CPT resonance [16]. It should be mentioned that the question of possibilities and limitations of SNR improvement within the proposed method necessitates a separate study, which will be carried out in the future.

The proposed method relies on measurement of direct radiation, thus ensuring relatively higher SNR as compared to measurement of scattered radiation or fluorescence. Therefore, the presented method holds a significant potential for application at lower pump radiation intensities and/or in shorter optical cells.

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